

MARKSCHEME

May 2003

MATHEMATICS

Higher Level

Paper 1

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Paper 1 Markscheme

Instructions to Examiners

Note: The number of marks for each question has been increased to 6. Where there are 2 marks (e.g. M2, A2) for an answer do NOT split the marks unless otherwise instructed.

1 Method of Marking

- (a) All marking must be done using a **red** pen.
- (b) In this paper, the maximum mark is awarded for a **correct answer**, irrespective of the method used. Thus, if the correct answer appears in the answer box, award the maximum mark and move onto the next question; in this case there is no need to check the method.
- (c) If an **answer is wrong**, then marks should be awarded for the method according to the markscheme. (A correct answer incorrectly transferred to the answer box is awarded the maximum mark.)

2 Abbreviations

The markscheme may make use of the following abbreviations:

- M** Marks awarded for **Method**
- A** Marks awarded for an **Answer** or for **Accuracy**
- G** Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown
- C** Marks awarded for **Correct** answers (irrespective of working shown)
- R** Marks awarded for clear **Reasoning**

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

- (i) penalize the error when it **first occurs**;
- (ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;
- (iii) award **M** marks for a correct method and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure.

| Markscheme | | Candidate's Script | Marking | |
|--|-----------|--|---------|---------------|
| $\$ 600 \times 1.02$ | <i>MI</i> | Amount earned = $\$ 600 \times 1.02$ | ✓ | <i>MI</i> |
| = $\$ 612$ | <i>AI</i> | = $\$ 602$ | × | <i>A0</i> |
| $\$ (306 \times 1.02) + (306 \times 1.04)$ | <i>MI</i> | Amount = $301 \times 1.02 + 301 \times 1.04$ | ✓ | <i>MI</i> |
| = $\$ 630.36$ | <i>AI</i> | = $\$ 620.06$ | ✓ | <i>AI(ft)</i> |

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

- (i) **fewer** marks should be awarded at the discretion of the Examiner;
- (ii) marks awarded should be followed by “(d)” (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

- (i) a mark should be awarded followed by “(d)” (to indicate that the marks have been awarded at the **discretion** of the Examiner);
- (ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1, METHOD 2, etc.** Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

$$\begin{aligned} \text{Mean} &= 7906/134 && \text{(M1)} \\ &= 59 && \text{(A1)} \end{aligned}$$

OR

$$\text{Mean} = 59 \qquad \qquad \qquad \text{(G2)}$$

- (b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.

On the markscheme, these equivalent numerical or algebraic forms will be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect *i.e.*, once the correct answer is seen, ignore further working.

- (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7, 1·7, 1,7 ; different forms of vector notation such as \vec{u} , \overline{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

There are two types of accuracy errors, incorrect level of accuracy, and rounding errors.

Unless the level of accuracy is specified in the question candidates should be penalized **once only IN THE PAPER** for any accuracy error (**AP**). This could be an incorrect level of accuracy (**only applies to fewer than three significant figures**), or a rounding error. Hence, on the **first** occasion in the paper when a correct answer is given to fewer than 3 significant figures, or rounded incorrectly, maximum marks are **not** awarded, but on **all subsequent occasions** when accuracy errors occur, then maximum marks **are** awarded.

(a) Level of accuracy

- (i) In the case when the accuracy of the answer is **specified in the question** (for example: “find the size of angle A to the nearest degree”) the maximum mark is awarded **only if** the correct answer is given to the accuracy required.
- (ii) When the accuracy is **not** specified in the question, then the general rule applies:

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

However, if candidates give their answers to more than three significant figures, this is acceptable.

(b) Rounding errors

Rounding errors should only be penalized at the **final answer** stage. This does **not** apply to intermediate answers, only those asked for as part of a question. Premature rounding which leads to incorrect answers should only be penalized at the answer stage.

Incorrect answers are wrong, and should not be considered under (a) or (b).

Examples

A question leads to the answer 4.6789....

- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy: 4.7 should be penalised the first time this type of error occurs, but 4.679 is **not** penalized, as it has more than three significant figures.
- 4.67 is incorrectly rounded – penalize on the first occurrence.
- 4.678 is incorrectly rounded, but has more than the required accuracy, do **not** penalize.

Note: All these “incorrect” answers may be assumed to come from 4.6789..., even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalized as being incorrect answers, not as examples of accuracy errors.

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks

Calculator penalties

Candidates are instructed to write the make and model of their calculator on the front cover. Please apply the following penalties where appropriate.

(i) Illegal calculators

If candidates note that they are using an illegal calculator, please report this on a PRF, and deduct 10 % of their overall mark. Note this on the front cover.

(ii) Calculator box not filled in.

Please apply a calculator penalty (**CP**) of 1 mark if this information is not provided. Note this on the front cover.

QUESTION 1

$$a(1+r) = 15; \frac{a}{1-r} = 27 \quad (A1)(A1)$$

$$1-r^2 = \frac{15}{27} \text{ or } a^2 - 54a + 405 = 0 \quad (M1)(A1)$$

(a) $r = \frac{2}{3}$ (A1) (C3)

(b) $a = 27 \times \frac{1}{3}$
 $a = 9$ (A1) (C3)

QUESTION 2

METHOD 1

$$2 \cos^2 \theta - 1 = 1 - \cos^2 \theta \text{ (or } 1 - 2 \sin^2 \theta = \sin^2 \theta) \quad (M1)(A1)$$

$$\Rightarrow 3 \cos^2 \theta = 2 \text{ (or } 3 \sin^2 \theta = 1) \quad (M1)$$

$$\cos \theta = \pm \sqrt{\frac{2}{3}} \left(\text{or } \sin \theta = \pm \sqrt{\frac{1}{3}} \right) \quad (A1)$$

$$\theta = 0.615, 2.53 \text{ (accept } 0.196\pi, 0.804\pi) \quad (A1)(A1) \quad (C3)(C3)$$

METHOD 2

$$\cos 2\theta = \frac{1}{2}(1 - \cos 2\theta) \quad (M1)(A1)$$

$$\cos 2\theta = \frac{1}{3} \quad (M1)(A1)$$

$$\theta = 0.615, 2.53 \text{ (accept } 0.196\pi, 0.804\pi) \quad (A1)(A1) \quad (C3)(C3)$$

Notes: Do not penalize if the candidate has included extra solutions.
 Penalize **[1 mark]** if candidates give answers in degrees, *i.e.*
 award **(A1)** for $35.3^\circ, 145^\circ$; **(A0)** for one correct answer in degrees.

QUESTION 3

METHOD 1

$$a \times b = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -3 & 2 & 2 \end{vmatrix} \quad (M1)(A1)$$

$$= i(4+2) + j(3-2) + k(2+6) \quad (M1)$$

$$= 6i + j + 8k \quad (A1)$$

$$(6i + j + 8k) \cdot (2i - 3j + 4k) = 41 \quad (M1)(A1) \quad (C6)$$

METHOD 2

$$(a \times b) \cdot c = \begin{vmatrix} 1 & 2 & -1 \\ -3 & 2 & 2 \\ 2 & -3 & 4 \end{vmatrix} \quad (M1)(A1)$$

$$= 1(8+6) - 2(-12-4) - 1(9-4) \quad (A1)(A1)(A1)$$

$$= 14 + 32 - 5 = 41 \quad (A1) \quad (C6)$$

QUESTION 4

Using the remainder and factor theorems, or long division,

$$8 + 4a - 6 + b = 0 \quad (M1)(A1)$$

$$-1 + a + 3 + b = 6 \quad (M1)(A1)$$

$$a = -2, b = 6 \quad (A1)(A1) \quad (C6)$$

Note: Award (C2) for one correct answer with no working.

QUESTION 5

$$\text{singular matrix} \Rightarrow \det = 0 \quad (R1)$$

$$A - \lambda I = \begin{vmatrix} 3 - \lambda & -2 \\ -3 & 4 - \lambda \end{vmatrix} \quad (A1)$$

$$(3 - \lambda)(4 - \lambda) - 6 = 0 \quad (M1)$$

$$\Rightarrow \lambda^2 - 7\lambda + 6 = 0 \quad (A1)$$

$$\lambda = 1 \text{ or } 6 \quad (A1)(A1) \quad (C6)$$

Note: Award (C2) for one correct answer with no working.

QUESTION 6

(a) X is $B(10, 0.25)$ (seen or implied) (R1)

so $E(X) = 10 \times 0.25 = 2.5$ (M1)(A1) (C3)

(b) $P(X \leq 2) = (0.75)^{10} + \binom{10}{1}(0.75)^9(0.25) + \binom{10}{2}(0.75)^8(0.25)^2$ (M1)(A1)

$= 0.526$ (A1) (C3)

QUESTION 7

- (a) $f_{\max} = 1.17$ (G2) (C2)
- (b) Roots are $-1.32, 0.537$ (G2)(G2) (C2)(C2)

QUESTION 8

$$\sin C = \frac{c \sin A}{a} = \frac{5 \times 0.5}{3} \quad (M1)(A1)$$

$$\hat{C} = 56.4^\circ \text{ or } 123.6^\circ \quad (A1)(A1)$$

$$\hat{B} = 93.6^\circ \text{ or } 26.4^\circ \quad (A1)(A1) \quad (C6)$$

Note: Award (C1) for one correct answer with no working.

QUESTION 9

(a) $0.88 = 0.4 + P(B) - 0.4P(B)$ (M1)(A1)
 $0.6P(B) = 0.48 \Rightarrow P(B) = 0.8$ (M1)(A1) (C4)

(b) **METHOD 1**

$$P(A \cup B) - P(A \cap B) = 0.88 - 0.32 \quad (M1)$$

$$= 0.56 \quad (A1) \quad (C2)$$

METHOD 2

$$P(A)P(B') + P(A')P(B) = 0.4 \times 0.2 + 0.6 \times 0.8 \quad (M1)$$

$$= 0.56 \quad (A1) \quad (C2)$$

QUESTION 10

METHOD 1

$$3x^2y^2 + x^3 2y \frac{dy}{dx} = 0 \quad (M1)(A1)$$

$$\text{At } (2, 1), 12 + 16 \frac{dy}{dx} = 0 \quad (M1)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{3}{4} \quad (A1)$$

$$\text{Gradient of normal} = \frac{4}{3} \quad (A1)$$

$$\text{Equation of normal is } y - 1 = \frac{4}{3}(x - 2) \quad (A1) \quad (C6)$$

METHOD 2

$$y = 2\sqrt{2}x^{\frac{3}{2}} \quad (A1)$$

$$\frac{dy}{dx} = -3\sqrt{2}x^{\frac{5}{2}} \quad (M1)(A1)$$

$$= -\frac{3}{4} \text{ when } x = 2 \quad (A1)$$

$$\text{Gradient of normal} = \frac{4}{3} \quad (A1)$$

$$\text{Equation of normal is } y - 1 = \frac{4}{3}(x - 2) \quad (A1) \quad (C6)$$

QUESTION 11

$$\sqrt{z} = \frac{2}{(1-i)} \cdot \frac{(1+i)}{(1+i)} + 1 - 4i \quad (M1)(A1)$$

$$= 1 + i + 1 - 4i \quad (A1)$$

$$= 2 - 3i \quad (A1)$$

$$z = (2 - 3i)^2 \quad (M1)$$

$$= -5 - 12i \quad (A1)$$

$$(\text{or } x = -5, y = -12) \quad (C3)(C3)$$

QUESTION 12

$$\text{Taking logs, } x \ln 3 + (2x + 1) \ln 4 = (x + 2) \ln 6 \quad (M1)(A1)$$

$$x(\ln 3 + 2 \ln 4 - \ln 6) = 2 \ln 6 - \ln 4 \quad (A1)$$

$$x = \frac{2 \ln 6 - \ln 4}{(\ln 3 + 2 \ln 4 - \ln 6)} \quad (A1)$$

$$= \frac{\ln 9}{\ln 8} \left(\text{Accept } \frac{\ln 81}{\ln 64} \text{ or equivalent} \right) \quad (M1)(A1)$$

$$(\text{or } a = 9, b = 8) \quad (C3)(C3)$$

QUESTION 13

METHOD 1

The graphs of $y = |x - 2|$ and $y = |2x + 1|$ meet where

$(x - 2) = (2x + 1) \Rightarrow x = -3$ (M1)(A1)

$(x - 2) = -(2x + 1) \Rightarrow x = \frac{1}{3}$ (M1)(A1)

Test any value, e.g. $x = 0$ satisfies inequality (M1)

so $x \in \left[-3, \frac{1}{3}\right]$. (A1) (C6)

METHOD 2

$(x - 2)^2 \geq (2x + 1)^2$ (M1)

$x^2 - 4x + 4 \geq 4x^2 + 4x + 1$ (A1)

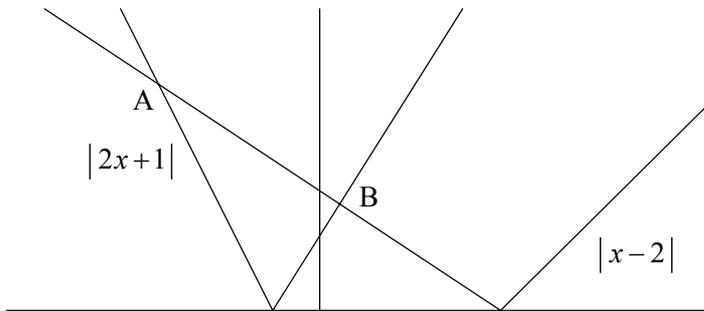
$3x^2 + 8x - 3 \leq 0$ (A1)

$(3x - 1)(x + 3) \leq 0$ (or find roots of equation) (A1)

Test any value, e.g. $x = 0$ satisfies inequality. (M1)

So $x \in \left[-3, \frac{1}{3}\right]$ (A1) (C6)

METHOD 3



(G1)

(G1)

We obtain for A, $x = -3$ and for B, $x = \frac{1}{3}$ (G1)(G1)

From the graph, $x \in \left[-3, \frac{1}{3}\right]$ (M1)(A1) (C6)

Note: Award (C5) for an open interval.

QUESTION 14

Let $E(X) = \mu$.

From tables, $z_1 = 0.44$, $z_2 = 1.53$ (A1)(A1)

$10 = \mu + 0.44\sigma$ (A1)

$12 = \mu + 1.53\sigma$ (A1)

$\mu = \frac{1.53 \times 10 - 0.44 \times 12}{1.53 - 0.44}$ (M1)

$\Rightarrow E(X) = 9.19$ (A1) (C6)

QUESTION 15

Equation of line is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ **(M1)(A1)**

Coordinates of foot satisfy

$2(1 + 2\lambda) + (1 + \lambda) - (9 - \lambda) = 6$ **(M1)(A1)**

$6\lambda = 12 \Rightarrow \lambda = 2$ **(A1)**

Foot of perpendicular is (5, 3, 7) **(A1) (C2)(C2)(C2)**

QUESTION 16

Distance travelled = $\int_0^\pi e^{-\sqrt{t}} \sin t dt + \left| \int_\pi^{2\pi} e^{-\sqrt{t}} \sin t dt \right|$

First integral = 0.620 **(G2)**

Second integral = $(\pm)0.232$ **(G2)**

Distance travelled = 0.852 **(G2) (C6)**

QUESTION 17

Let $y = \frac{x^2 - 1}{x^2 + 1} \Rightarrow yx^2 + y = x^2 - 1$ **(M1)(A1)**

$x^2(1 - y) = 1 + y \Rightarrow x^2 = \frac{1 + y}{1 - y}$ **(M1)(A1)**

$x = \pm \sqrt{\frac{1 + y}{1 - y}}$

Interchanging, $y = \pm \sqrt{\frac{1 + x}{1 - x}}$ **(A1)**

$f^{-1}(x) = -\sqrt{\frac{1 + x}{1 - x}}$ **(A1) (C6)**

QUESTION 18

$I = \int \left(\frac{2 - y}{y} \right)^2 (-dy)$ **(M1)(A1)**

$= -\int \left(\frac{4}{y^2} - \frac{4}{y} + 1 \right) dy$

$= \frac{4}{y} + 4 \ln|y| - y + c$ **(A1)(A1)(A1)**

$= \frac{4}{2 - x} + 4 \ln|2 - x| - (2 - x) + c$ **(A1) (C6)**

Note: c and modulus signs not required.

QUESTION 19

(a) $\bar{x} = \frac{626}{20} = 31.3$ (M1)(A1) (C2)

(b) **METHOD 1**

$$\begin{aligned} \sum (x - \bar{x})^2 &= \sum x^2 - 2\bar{x} \sum x + \sum \bar{x}^2 && (M1) \\ &= 19780.8 - 2 \times 31.3 \times 626 + 20 \times 31.3^2 && (A1) \\ &= 187 && (A1) \end{aligned}$$

Unbiased estimate = $\frac{187}{19} = 9.84$ (A1) (C4)

METHOD 2

$$\begin{aligned} \text{Unbiased estimate} &= \frac{\sum x^2}{n-1} - \frac{(\sum x)^2}{n(n-1)} && (M1) \\ &= \frac{19780.8}{19} - \frac{626^2}{19 \times 20} && (A1)(A1) \\ &= 9.84 && (A1) \end{aligned}$$

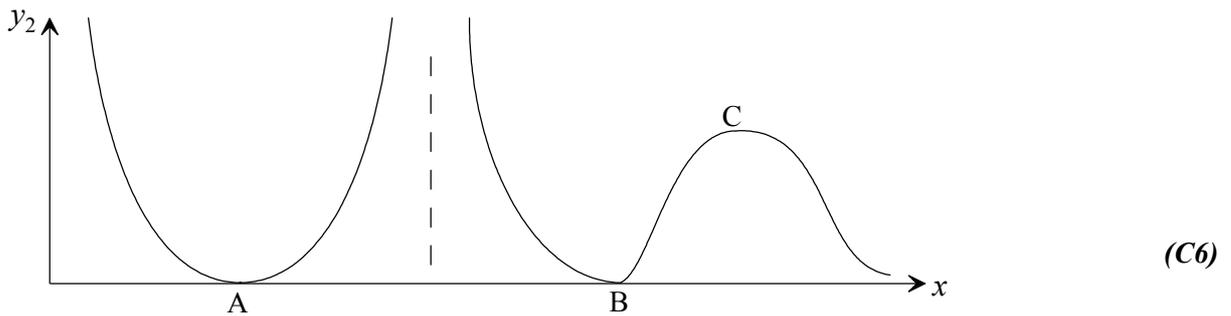
(C4)

METHOD 3

$$\begin{aligned} \sum \frac{x^2}{n} - \bar{x}^2 &= \frac{19780.8}{20} - 31.3^2 && (M1)(A1) \\ &= 9.35 && (A1) \end{aligned}$$

Unbiased estimate = $9.35 \times \frac{20}{19} = 9.84$ (A1) (C4)

QUESTION 20



Notes: Award (A1) for zero at A, (A1) for correct (concave) shape to left of dotted line, (A1) for correct (concave) descent to right of dotted line, (A1) for zero at B, (A1) for maximum at C, (A1) for asymptotic to x-axis as $x \rightarrow \infty$. Please note that the first and fourth (A1) marks are given for the candidate's graph hitting the x-axis at A and B. No marks are given for the exact shape of this graph at A and B since it is not possible to deduce from the given graph whether or not the gradient of y_2 is continuous at these points.